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Math 12 Honours: Section 5.1 Solving Exponential Functions

1. Solve for all values of "x" :

a) $8^{x+1}(32)=128$ $8^{x+1}=4$ $2^{3(x+1)}=2^2$ $3(x+1)=2$ $x+1=\frac{2}{3}$ $\boxed{x=-\frac{1}{3}}$	b) $\left(27^{2x}\right)^3=\frac{1}{729}$ $729^{3x}=729^{-1}$ $3x=-1$ $\boxed{x=-\frac{1}{3}}$
c) $\frac{8^{-1}+2^{-3}}{64^x}=\left(2^{2x-3}\right)$ $\frac{2^{-3}+2^{-3}}{2^{6x}}=2^{2x-3}$ $2^{-2-6x}=2^{2x-3}$ $-2-6x=2x-3$ $8x=1$ $\boxed{x=\frac{1}{8}}$	d) $\frac{256^x}{4^{2x-6}}=\left(2^{2x}\right)^x$ $\frac{2^{8x}}{2^{4x-12}}=\left(2^{2x}\right)^x$ $2^{8x-4x+12}=2^{2x^2}$ $4x+12=2x^2$ $2x^2-4x-12=0$ $x=\frac{4\pm\sqrt{16+8\cdot 12}}{4}$ $x=\frac{4\pm 4\sqrt{7}}{4}$ $\boxed{x=1\pm\sqrt{7}}$
e) $\left(8^{x+4}\right)^x=128^{2x+3}$ $\left(2^{3x+12}\right)^x=2^{14x+21}$ $3x^2+12x=14x+21$ $3x^2-2x-21=0$ $1 \quad -3$ $3 \quad 7$ $(x-3)(3x+7)=0$ $\boxed{x=3 \quad x=-\frac{7}{3}}$	f) $\frac{\left(729^x\right)}{3\times 9^{2x-3}}=\left(27^{2x+3}\right)^x$ $\frac{3^{6x}}{3^{4x-6+1}}=3^{(6x+9)x}$ $6x-4x+5=6x^2+9x$ $6x^2+7x-5=0$ $\frac{2}{3} \quad \frac{-1}{5}$ $(2x-1)(3x+5)=0$ $\boxed{x=\frac{1}{2}}$ $\boxed{x=-\frac{5}{3}}$

2. Use Logarithms to solve for "x":

a) $4^{x-3}=10$ $\frac{4^x}{4^3}=10 \Rightarrow 4^x=640 \Rightarrow x=\frac{\log 640}{\log 4}=\log_4 640$ $=\boxed{4.66}$	b) $6^{2x+1}=14$ $2x+1=\log_6 14 \Rightarrow x=\frac{\log_6 14 - 1}{2}=\boxed{0.236}$
c) $9^{x^2}=20$ $x^2=\log_9 20 \Rightarrow x=\pm\sqrt{\log_9 20}=\boxed{\pm 1.168}$	d) $10^{3-x}=21$ $3-x=\log 21 \Rightarrow x=3-\log 21$ $\boxed{x=1.678}$
e) $(2^{2x})3^x=8000$ $(4\times 3)^x=8000$ $x=\log_{12} 8000=\boxed{3.62}$	f) $6^{x^3}=20$ $x=\sqrt[3]{\log_6 20}=\boxed{1.187}$

3. Rewrite each of the following in logarithm form:

$a)\ e^5=y$ $\boxed{5=\log_e y=\frac{\log y}{\log e}}$	$b)\ 2a^b=c$ $b=\log_a \frac{c}{2}$	$c)\ 2(3x)^{15}=y$ $\boxed{15=\log_{3x}\frac{y}{2}}$

4. Evaluate each of the following without a calculator:

$a)\log_5 125$ $=\boxed{3}$	$b)\log_2 128$ $=\boxed{7}$	$c)\log_9 2183$ $\log_9 3^7=7\log_9 3=7\cdot\frac{1}{2}=\boxed{\frac{7}{2}}$
$d)\log_{16} 64$ $\log_{2^4} 2^6=\log_{2^4} 2^4+\log_{2^4} 2^2$ $=1+\frac{1}{2}=\boxed{\frac{3}{2}}$	$e)\log_7 \sqrt{343}$ $\log_7 7^{\frac{3}{2}}=\boxed{\frac{3}{2}}$	$f)(\log_{128} 1024)^{-1}$ $(\log_{2^7} 2^{10})^{-1}=(1+\frac{3}{7})^{-1}=(\frac{10}{7})^{-1}=\boxed{\frac{7}{10}}$
$g)\log_8\left(\frac{1}{4}\right)$ $\log_{2^3} 2^{-2}=\boxed{\frac{-2}{3}}$	$h)-\log_{2.5}\left(\frac{125}{8}\right)$ $-\log_{\frac{5}{2}}\left(\frac{5}{2}\right)^3=\boxed{-3}$	$i)\left(\log_{0.125} 64^{\frac{2}{3}}\right)^{-2}$ $(\log_{2^{-3}} 2^4)^{-2}=\left(\frac{4}{-3}\right)^{-2}=\boxed{\frac{9}{16}}$

5. Simplify each of the following logarithms without using a calculator:

a) $\log_6 24+\log_6 9$ $\log_6 24\cdot 9=\log_6 6^3=\boxed{3}$	b) $\log_5 100-\log_5 4$ $\log_5 \frac{100}{4}=\log_5 25=\boxed{2}$	c) $\log_4 8+\log_4 64$ $\log_4 8\times 64=\log_{2^2} 2^9=\boxed{\frac{9}{2}}$

d) $\log_3 324 - \log_3 4$ $\log_3 \frac{324}{4} = \log_3 81 = \boxed{4}$	e) $\log_4 12 + \log_4\left(\frac{2}{3}\right) + 3\log_4 2$ $\log_4 \left(12\times\frac{2}{3}\times 2^3\right) = \log_4 2^6 = \boxed{3}$	f) $\log_8 24 + 2\log_8 2 - \log_8 3$ $\log_8 \frac{24\times 2^2}{3} = \log_{2^3} 2^5 = \boxed{\frac{5}{3}}$
g) $\log_3 \sqrt{45} - 0.5\log_3 5 + \log_3 9$ $\log_3 \frac{\sqrt{45}}{\sqrt{5}}\cdot 9 = \log_3 27 = \boxed{3}$	h) $\log_3 5! - \log_3 4 - 1$ $= \log_3 5! - \log_3 4 - \log_3 3$ $= \log_3 \frac{5!}{4\times 3} = \boxed{\log_3 10} = \boxed{2.096}$	i) $\log_{x+1}\left(\underbrace{x^3+3x^2+3x+1}_{\text{binomial expansion}}\right)$ $= \log_{x+1}(x+1)^3 = \boxed{3}$

6. Solve for all possible values of "x" : Euclid : $3^{x-1}\left(9^{\frac{3}{2x^2}}\right) = 27$

$$3^{x-1+\frac{3}{x^2}} = 3^3$$

$$x-1+\frac{3}{x^2} = 3$$

$$x+\frac{3}{x^2} = 4$$

$$x^3+3 = 4x^2$$

$$x^3-4x^2+3 = (x-1)(x^2-3x-3)$$

$$\begin{array}{r} x^2-3x-3 \\ x-1\;\overline{)\,x^3-4x^2+3} \\ -(x^3-x^2) \\ \hline -3x^2+3 \\ -(-3x^2+3x) \\ \hline -3x+3 \end{array}$$

$$\boxed{x = 1,\ \frac{3\pm\sqrt{21}}{2}}$$

7. Suppose you have the equation $2^{5x-3} = -1$, is it possible to have a solution? Explain and justify your answer:

No, 2 to the power of nothing can be negative!

Thus we cannot log a negative number.

$$5x-3 = \log_2 -1 = \varnothing$$

8. Given the equation, $m^x = m^y$, how many cases are there such that the equation is true?

① $x=y$

② $x, y = 1$

③ $x, y = 0$

④ $m = 1$

⑤ $m=-1$ and x, y are both either odd or even.

9. Find all values of "x" such that: $6^2\left(6^x\right)^x = \left(6^x\right)\left(6^x\right)\left(6^x\right)$

$$6^{2+x^2} = 6^{3x}$$

$$2+x^2 = 3x$$

$$x^2-3x+2=0$$

$$(x-1)(x-2)=0$$

$$\boxed{x_1=1} \quad \boxed{x_2=2}$$

10. Solve for all possible values of "x": $4\times 9^x - 21 = 25\times 3^x$

$4\times 3^{2x} - 25\times 3^x - 21 = 0$

$3^x = A$

$4A^2 - 25A - 21 = 0$

$(A-7)(4A+3)=0$

$A=7$ $\qquad$ $A=-\frac{3}{4}$ rej.

$\boxed{x=\log_3 7}$ $\qquad$ $\cancel{x=\log_3 -\frac{3}{4}}$

11. Solve for "x": $2^{x+1} + 2^x = 2^3 + 2^5 + 2^{7-x}$

$2^{2x+1} + 2^{2x} = 2^{x+3} + 2^{x+5} + 2^7$

$2^{2x}(3) - 2^x(40) - 128 = 0$

$2^x = \frac{40 \pm \sqrt{40^2+4\cdot 3(128)}}{6} = \cancel{-2.6}, 16$ rej.

$2^x = 16 \Rightarrow \boxed{x=4}$

12. Suppose that $P = 2^m$ and $Q = 3^n$. Which of the following is equal to 12^{mn} for every pair of integers (m,n)?

a) P^2Q b) P^nQ^m c) P^nQ^{2m} d) $P^{2m}Q^n$ e) $P^{2n}Q^m$ (AMC)

$P^{2n} = 4^{mn}$

$Q^m = 3^{nm}$ $\Rightarrow P^{2n}Q^m = 12^{mn}$

13. Solve the equation for "y" in terms of "x". For ~~hat~~ what values of "x" is there no value of "y" that satisfies the equation: $(x+2)^{2y} - x^2 = 3$

$2y = \log_{(x+2)}(3+x^2)$

$\boxed{y = \frac{\log_{x+2} 3+x^2}{2} = \log_{x+2}\sqrt{3+x^2}}$

Restrictions:

$x+2>0$ $\qquad$ $x+2\neq 1$

$\boxed{x>-2 \qquad x\neq -1}$

$\sqrt{3+x^2}>0$ guaranteed

14. Suppose that $60^a = 3$ and $60^b = 5$. What is the value of $12^{\frac{1-a-b}{2-2b}}$? AMC12

$60^{a+b} = 15 \Rightarrow 60^{1-a-b} = \frac{60}{60^{a+b}} = \frac{60}{15} = 4$

$60^{2b} = 25 \Rightarrow 60^{2-2b} = \frac{60^2}{60^{2b}} = \frac{60\times 60}{25} = 144$

$\frac{1-a-b}{2-2b} = \frac{\log_{60} 4}{\log_{60} 144}$

$\frac{1-a-b}{2-2b} = \log_{144} 4 = \log_{12} 2$

$12^{\frac{1-a-b}{2-2b}} = 12^{\log_{12} 2}$

$= \boxed{2}$

15. Suppose "a", "b", "c" and "d" are positive integers, then what is the smallest possible value of $a+b+c+d$?

$4^a + 4^b + 4^c + 4^d = 4^{1023}$

$2^a + 2^b = 2^c$

have to be equal

$a=b=c=d$

so

$4(4^a) = 4^{1023} \Rightarrow a = 1022 \Rightarrow a+b+c+d = \boxed{4088}$

16. Given the equations below, find all the ordered triples of real values (a,b,c) that satisfy the them:

$c^a = b^{2a}$, $2^c = 2(4^a)$, $a+b+c=10$

$2^c = 2^{2a+1}$

$c = 2a+1$

$3a+b=9 \Rightarrow 9-3a=b$

① $(2a+1)^a = (9-3a)^{2a}$

$2a+1 = (9-3a)^2$

$2a+1 = 81-54a+9a^2$

$9a^2-56a+80=0$

$1 \quad -4$

$9 \quad -20$

$(a-4)(9a-20)=0$

$a_1 = 4$ $a_2 = \frac{20}{9}$

② $9-3a=b$

$b_1 = 9-3(4)$

$b_1 = -3$

$b_2 = 9-3\left(\frac{20}{9}\right)$

$b_2 = 9-\frac{20}{3}$

$b_2 = \frac{7}{3}$

③ $c = 2a+1$

$c_1 = 2(4)+1$

$c_1 = 9$

$c_2 = 2\left(\frac{20}{9}\right)+1$

$c_2 = \frac{49}{9}$

$$\boxed{(a,b,c) = (4,-3,9), \left(\frac{20}{9}, \frac{7}{3}, \frac{49}{9}\right)}$$